Lattice study of $N_f = 10$ Yang-Mills theory

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Abstract

There exists a range of N_f over which Yang-Mills theory does not confine, instead exhibiting conformal behavior in the infrared. Non-perturbative studies are imperative to fix the lower end of this window; prior work [1] indicates that for $SU(3)$ Yang-Mills, the transition from confinement to conformal behavior is given by $8 < N_f^c < 12$. Here we propose a direct calculation of the running coupling with $N_f = 10$, to constrain further the value of N_f^c. If $N_f = 10$ is found to lie just outside the conformal window, this calculation may also lead to the first example of a "walking" theory simulated on the lattice. The simulation will be carried out in the Schrödinger functional formalism, with $O(a)$-improved Wilson fermions. This is a class B proposal, requesting $500,000$ 6n-equivalent node-hours.

I. PHYSICS GOALS

With the Large Hadron Collider (LHC) about to come online, the bulk of particle theory research is focused on extending the Standard Model to the TeV-scale energies which will soon become experimentally accessible. A number of the theoretical models considered to describe LHC physics involve new *strong dynamics*, i.e. new interactions resembling the only known strong force, QCD. Models such as technicolor [2, 3], composite Higgs [4] and topcolor [5] are just a few examples of Standard Model extensions involving new strong interactions.

If another strongly interacting theory does appear in nature, lattice gauge theory will provide the ideal way to study it. Perturbation theory can provide only a limited understanding of a strongly coupled theory, and no other non-perturbative method is as mature and broadly applicable as lattice simulation.

Even if no strongly coupled dynamics are revealed at the LHC, the study of more exotic Yang-Mills theories can be interesting in and of itself. Mapping out the parameter space of general Yang-Mills theories by varying the number of colors and light flavors would lead to a more solid theoretical understanding, which might open up new avenues of non-perturbative investigation into QCD or any strongly-coupled theory that might occur beyond the LHC.

In particular, our goal in this project is to study the dependence of the properties of $SU(3)$ Yang-Mills theory on the number of light dynamical flavors, N_f. Two of the most important features of QCD ($N_f = 2$ at relatively low energies) are *asymptotic freedom* (the coupling strength vanishes at high energy/short distance) and *confinement* (the coupling strength diverges at low energy/long distance, i.e. there are no free states with color charge.) These properties are strongly dependent on N_f; it is well known that for $N_f > 16.5$, SU(3) Yang-Mills is no longer asymptotically free [6]. Since a lattice simulation generally requires asymptotic freedom in order to take the continuum limit, we will not discuss these theories here. For N_f just below this critical value, the short-distance theory still resembles QCD, but in the infrared the coupling flows to a perturbative fixed point [7] - confinement is lost.

Clearly as we decrease the number of flavors, at some critical point $2 < N_f^c < 16.5$ a transition must take place from a fixed-point theory to a confining theory. Since the theory for $N_f > N_f^c$ shows conformal behavior in the infrared, we refer to the range $N_f^c < N_f < 16.5$ as the *conformal window*. Although perturbation theory is useful near the top of the

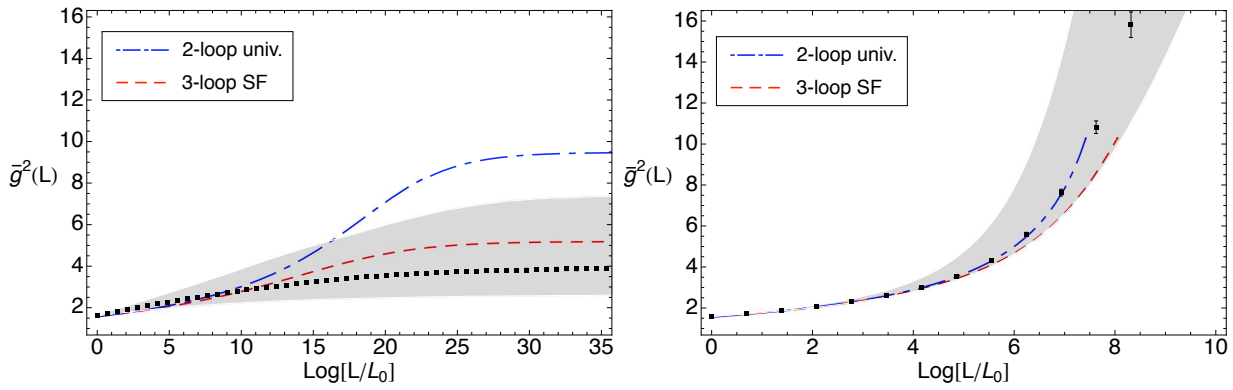

FIG. 1: Running coupling at $N_f = 12$ (left) and $N_f = 8$ (right) in $SU(3)$ Yang-Mills, as determined in [1]. The lack of an observed fixed point at 8 flavors indicates that this value of N_f lies outside the conformal window.

conformal window, as we decrease N_f the strength of the fixed-point coupling increases, eventually making the perturbative expansion useless. Non-perturbative study is essential to accurately determine the value of N_f^c.

Our prior work [1] has shown evidence that $8 < N_f^c < 12$. Our purpose in simulating at $N_f = 10$ is to continue this study of the conformal window; specifically, we have three primary goals. First, this project will allow us to further constrain the lower end of the conformal window. Moreover, comparisons of $N_f = 10$ simulation results with those obtained at 8 and 12 flavors, along with QCD and other multi-flavor results [8] may shed some light on the nature of the transition between conformal and confining theories itself. Finally, if $N_f = 10$ is discovered to be just outside the conformal window, there is some hope that it will show conformal behavior over an intermediate range of energy scales before finally confining in the infrared, a behavior known as *walking* (since the "running" evolution of the coupling is slowed down.) Walking theories can open up a number of new possibilities in the construction of models with strong dynamics, and are of particular interest in the context of technicolor [9].

Finally, the possibility that $N_f = 10$ lies in the conformal window and thus exhibits an IR fixed point is also of interest, in the context of research into conformal field theory, e.g. [10, 11]. Although not truly conformal, theories inside the conformal window do show approximate scale invariance, and the ability to simulate such a theory on the lattice may offer new ways to study conformal behavior non-perturbatively. Furthermore, quantities

derived from our running coupling measurement, such as the fixed point coupling strength and the anomalous scaling dimension at the fixed point, may be of interest to model builders thinking about conformal behavior.

II. COMPUTATION

The Schrödinger functional (SF) approach is based on a nonstandard set of boundary conditions, with periodic boundary conditions in all spatial directions but Dirichlet boundaries in the temporal direction. It provides an alternative to the Wilson loop method for measuring the running coupling strength, one which is free from finite-size effects since the coupling is measured at the scale of the box, L. The presence of the box also lifts the zero modes of the Dirac operator, making it possible to simulate directly at zero fermion mass [12].

In particular, for the running coupling measurement the boundary gauge field values are set to be consistent with the classical solution for a constant chromoelectric background field, with strength parameterized by a dimensionless value η. The coupling is then measured to be inversely proportional to the response of the action as the strength parameter η is varied,

$$\frac{dS}{d\eta} \equiv \frac{k}{\overline{g}^2}. \tag{1}$$

where k is a constant of proportionality, which is set so that the observable $\overline{g}^2$ matches onto the perturbative running coupling.

We propose to use the standard Wilson gauge action along with the clover-improved Wilson fermion action for this simulation. Gauge configurations will be evolved using the hybrid Monte Carlo (HMC) algorithm. Non-perturbative tuning will be performed to locate the massless point and to determine the appropriate value of the clover coefficient. This will remove all $O(a)$ artifacts in the bulk; however, the presence of the Dirichlet boundaries introduces additional $O(a)$ terms into the action which must be dealt with. We will also utilize perturbative boundary counterterms for this project, as detailed below.

Since the presence of SF boundary conditions lifts the zero modes of the Dirac operator, we can avoid having to perform a chiral extrapolation by simulating directly at zero current fermion mass. The current mass is defined by way of the PCAC relation [13],

$$\partial_\mu A^a_\mu = 2mP^a \tag{2}$$

where A_μ^a and P^a are the axial current and pseudoscalar current, respectively. In order to simulate at zero mass, non-perturbative tuning will be required to locate the critical line $m_c(\beta)$. The value of the clover coefficient c_{sw} must also be tuned to remove $O(a)$ artifacts; these tunings can be done simultaneously in a single set of simulations. Since the mass term and clover term are local operators in the bulk, they are expected to be fairly insensitive to the box size L. This means that the fine-tuning can be performed on lattices with relatively small a/L, at negligible computational cost compared to the rest of the project.

Even after the clover coefficient has been non-perturbatively determined, some $O(a)$ boundary effects remain in the SF action, due to additional operators induced by the presence of the Dirichlet boundaries. These artifacts appear in both the gauge and fermion action, and can be removed by introducing counterterms into the action, as detailed in [14]. We intend to use perturbatively computed counterterms in this simulation; the gauge action counterterms are known to two loops in lattice perturbation theory, while the fermionic counterterms are known to one loop [15].

In order to measure the running of the coupling over a large range of scales, we will employ the step scaling method, which is explained in detail in [12]. The basic idea is to match lattice calculations at different values of L/a, by tuning the lattice coupling β so that the coupling strength $\overline{g}^2(L)$ is equal on each lattice. In other words, we can match a simulation at large L/a onto one at small L/a using this procedure, and then we can "step" from $L/a \to sL/a$ while keeping the lattice spacing fixed, yielding $\overline{g}^2(sL)$. Iterating this procedure k times allows us to run the coupling from $\overline{g}^2(L)$ to $\overline{g}^2(s^kL)$, without requiring a large range of lattice sizes L/a (which would quickly become prohibitively expensive).

The SF running coupling is a difficult observable to measure, possessing very long autocorrelation times; since the observable is on the scale of the box, we measure only one statistically independent sample per gauge configuration, as opposed to an observable with a shorter correlation length which can yield many samples per lattice. At stronger couplings, the autocorrelation time is enhanced by the phenomenon of "excursions" in the time series of $\overline{g}^2$, noted in both our prior work [1] and by the ALPHA collaboration [16]. Due to the long autocorrelations, a very large number of Monte Carlo trajectories are required to obtain an accurate measurement at strong couplings, generally in excess of $80,000$.

III. SOFTWARE

For this calculation we intend to employ a customized version of the Chroma code package [17]. Chroma provides a well-tested code base, with many readily available optimizations such as the Hasenbusch trick [18], even-odd preconditioning, and a number of code optimizations (e.g. [19].)

The default Chroma code contains all the necessary routines for simulating clover improved Wilson fermions with Schrödinger functional boundary conditions. However, to our knowledge it has never been used for a running coupling measurement, and so some necessary routines are not included, namely the coupling measurement itself and an action containing the boundary improvement coefficients detailed above. These additions are currently under development, with the eventual intent of folding them into the main Chroma release. All of the changes required are relatively minor, and should not have a significant impact on the performance of the code.

Extensive analysis code has already been developed for our prior studies of the running coupling at 8 and 12 flavors [1], and would be employed for this project as well, requiring only trivial modifications.

IV. RESOURCES

To accurately measure the running coupling via step scaling, we will need to generate a large table of values $\overline{g}^2(\beta, L)$, to which we fit an interpolating function. In order to account for the total cost of this project, we need to determine the range $(\beta_{min}, \beta_{max})$ and the set of L/a values over which we will collect data, and the spacing in β of each individual measurement.

Interpolating on our prior observations at $N_f = 8$ and $N_f = 12$, we expect to require $\beta_{max} \sim 4.3$ at 10 flavors in order to observe coupling strengths on the order of $\overline{g}^2 \sim 20$, which satisfies most estimates of the strength required to break chiral symmetry (as indicated in [1]), and should thus exceed the value of any possible infrared fixed point. The correspondence between β and $\overline{g}^2$ is different for Wilson and staggered fermions, of course, but the difference is not expected to be larger than ~ 0.1; therefore as a worst case we assume $\beta_{max} = 4.2$ to compute our cost.

As noted above, in our previous work we found $40,000$ MD trajectories to be sufficient to remove autocorrelations at relatively weak couplings, with more required at larger $\overline{g}^2$/lower β. A simple way to account for this variation is to choose a value of β that divides "weak" from "strong" coupling simulations, and to set the target of $80,000$ trajectories on the strong side. Again based on our prior work, we estimate that this division line should be placed at $\beta \sim 5.0$.

The choice of the lower limit β_{min} is not crucial, since weak-coupling simulations will account for a small percentage of the total project cost. Based once again on intuition gained from our prior work, we will set $\beta_{min} = 7.0$ for this estimate.

The number of points entering into the continuum extrapolation of the step scaling function is determined by the number of pairs of data sets $L/a, 2L/a$ available. We propose the range of values $L/a = \{4, 5, 6, 8, 10, 12, 16\}$ for this study, yielding a four-point continuum extrapolation.

Finally, the spacing $\Delta\beta$ of our sampling in β must be determined. Since the cost of generating configurations increases sharply with L/a, we will have to space our measurements at the largest L/a relatively sparsely. We propose the following gradation:

$$\begin{cases} L/a = 4, 5, 6: & \Delta\beta = 0.05(\beta > 5.0), 0.1(\beta < 5.0) \\ L/a = 8, 10: & \Delta\beta = 0.1(\beta > 5.0), 0.2(\beta < 5.0) \\ L/a = 12, 16: & \Delta\beta = 0.2(\beta > 5.0), 0.4(\beta < 5.0) \end{cases}$$

where again, we intend to gather $40,000$ trajectories for $5.0 < \beta \leq 7.0$, and $80,000$ for $4.2 \leq \beta \leq 5.0$.

Timing information was gathered on the Kaon server at Fermilab. A Kaon node consists of two dual-core Opteron processors, with 2.0GHz clock speed. With SSE optimization enabled, average measured CG inverter performance was ~ 1 GFlop/core-sec. Timing information in trajectories per core-hour is shown in Table I.

With all of the details outlined above, the total cost of our simulations for each L/a is as shown in Table I. The total production cost is estimated to be 1,035,240 Kaon core-hours, or 454,730 6n-equivalent node hours. We request 500,000 node hours for this project; the balance will be used in non-perturbative tuning and to compensate for any unforeseen difficulties (job failures due to bad nodes, etc.)

L/a	traj/c-h ($\beta > 5.0$)	traj/c-h ($\beta < 5.0$)	c-h ($\beta > 5.0$)	c-h ($\beta < 5.0$)	total core-hours (Kaon)
4	702	446	1140	3050	4190
5	320	329	2500	4130	6630
6	145	162	2760	4440	7200
8	24.3	30.2	16460	23840	40300
10	8.1	10.2	49380	70580	119960
12	2.49	3.30	64260	72730	136990
16	0.49	0.61	326530	393440	719970

TABLE I: Timings and estimated total cost for the running coupling measurement at each L/a. "c-h" stands for "core-hour."

V. DATA SHARING AND EXCLUSIVITY

No propagators will be computed, and we do not intend to save gauge configurations to disk except for checkpointing use. The lattices we use have boundary conditions specifically tuned for the running coupling measurement, and so they would not be useful for other projects. Results of our non-perturbative tuning, which would be useful for other simulations using 10 flavors of clover fermions, will be made available.

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